

JEE-MAIN EXAMINATION – JANUARY 2026

(HELD ON WEDNESDAY 28th JANUARY 2026)

TIME : 9:00 AM TO 12:00 NOON

MATHEMATICS

TEST PAPER WITH SOLUTION

SECTION-A

1. If $g(x) = 3x^2 + 2x - 3$, $f(0) = -3$ and $4g(f(x)) = 3x^2 - 32x + 72$, then $f(g(2))$ is equal to:

- (1) $\frac{25}{6}$ (2) $-\frac{25}{6}$
(3) $\frac{7}{2}$ (4) $-\frac{7}{2}$

Ans. (3)

Sol. $g(2) = 13$

$$f(g(2)) = f(13)$$

$$\text{Now } 4g(f(x)) = 3x^2 - 32x + 72$$

$$4[3f^2(x) + 2f(x) - 3] = 3x^2 - 32x + 72$$

$$\text{Let } f(x) = t$$

$$12t^2 + 8t - (3x^2 - 32x + 72) = 0$$

$$f(x) = \frac{-8 \pm \sqrt{64 + 48(3x^2 - 32x + 72)}}{24}$$

$$f(x) = \frac{-8 \pm 4(3x - 16)}{24}$$

$$\therefore f(0) = -3 \therefore \text{we take +ve sign}$$

$$\therefore f(x) = \frac{-8 + 4(3x - 16)}{24}$$

$$\therefore f(13) = \frac{-8 + 4 \times 23}{24} = \frac{84}{24} = \frac{7}{2}$$

2. The value of $\sum_{k=1}^{\infty} (-1)^{k+1} \left(\frac{k(k+1)}{k!} \right)$ is :

- (1) $2/e$ (2) $1/e$
(3) $\sqrt{e}$ (4) $e/2$

Ans. (2)

Sol. $T_k = (-1)^{k+1} \cdot \frac{k(k+1)}{k!} = (-1)^{k+1} \left(\frac{k(k-1) + 2k}{k!} \right)$

$$\therefore \text{sum} = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k-2} + \sum_{k=1}^{\infty} \frac{2(-1)^{k+1}}{k-1}$$

$$= \left(\frac{1}{-1} - \frac{1}{0} + \frac{1}{1} - \frac{1}{2} + \frac{1}{3} \dots \right) + \left(\frac{2}{0} - \frac{2}{1} + \frac{2}{2} - \frac{2}{3} \dots \right)$$

$$= \frac{1}{e}$$

3. Let $y = x$ be the equation of a chord of the circle C_1 (in the closed half-plane $x \geq 0$) of diameter 10 passing through the origin. Let C_2 be another circle described on the given chord as its diameter. If the equation of the chord of the circle C_2 , which passes through the point $(2, 3)$ and is farthest from the center of C_2 , is $x + ay + b = 0$, then $a - b$ is equal to :

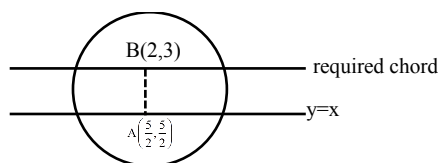
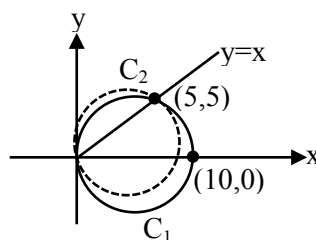
- (1) 10 (2) -6
(3) -2 (4) 6

Ans. (3)

Sol. Equation of circle C_2 is

$$x^2 + y^2 - 5x - 5y = 0$$

$$\text{its centre is } \left(\frac{5}{2}, \frac{5}{2} \right)$$



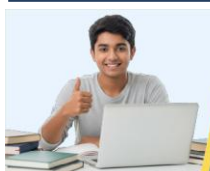
$$m_{AB} = -1$$

$$\therefore \text{Slope of required chord} = 1$$

$$\therefore \text{equation of required chord is } x - y + 1 = 0$$

$$\therefore a = -1, b = 2$$

$$\therefore a - b = -2$$



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4. If $\frac{\tan(A-B)}{\tan A} + \frac{\sin^2 C}{\sin^2 A} = 1$, $A, B, C \in \left(0, \frac{\pi}{2}\right)$,

then

- (1) $\tan A, \tan C, \tan B$ are in G.P.
- (2) $\tan A, \tan B, \tan C$ are in G.P.
- (3) $\tan A, \tan C, \tan B$ are in A.P.
- (4) $\tan A, \tan B, \tan C$ are in A.P.

Ans. (1)

Sol. $\frac{\tan A - \tan B}{(1 + \tan A \tan B) \tan A} + \frac{1 + \cot^2 A}{1 + \cot^2 C} = 1$

Put $\tan A = x, \tan B = y, \tan C = z$

$$\therefore \frac{x - y}{(1 + xy)x} + \frac{(x^2 + 1)z^2}{x^2(z^2 + 1)} = 1$$

$$\therefore x(x - y)(z^2 + 1) + z^2(1 + x^2)(1 + xy) = (1 + xy)x^2(1 + z^2)$$

after solving we get

$$z^2 = xy \quad \therefore 1 + x^2 \neq 0$$

$$\therefore \tan^2 C = \tan A \cdot \tan B$$

$\tan A, \tan C, \tan B$ are in G.P.

5. Let z be a complex number such that $|z - 6| = 5$ and $|z + 2 - 6i| = 5$.

Then the value of $z^3 + 3z^2 - 15z + 141$ is equal to

- (1) 42
- (2) 37
- (3) 50
- (4) 61

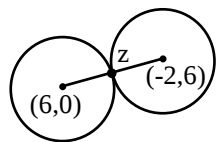
Ans. (3)

Sol. Center of first circle $C_1(6,0), r_1 = 5$

Center of second circle $C_2(-2,6), r_2 = 5$

$$\therefore C_1C_2 = r_1 + r_2$$

$\therefore$ common point Z is mid point of C_1 & C_2



$$\therefore z = 2 + 3i$$

$$\therefore z^2 = 4z - 13$$

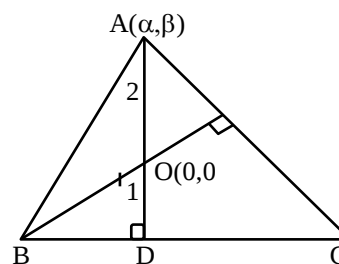
$$\therefore z^3 = 3z - 52$$

$$\therefore z^3 + 3z^2 - 15z + 141 = 50$$

6. Let ABC be an equilateral triangle with orthocenter at the origin and the side BC on the line $x + 2\sqrt{2}y = 4$. If the co-ordinates of the vertex A are (α, β) , then the greatest integer less than or equal to $|\alpha + \sqrt{2}\beta|$ is

- (1) 2
- (2) 3
- (3) 5
- (4) 4

Ans. (4)



Sol.

$$\therefore m_{BC} \cdot m_{AD} = -1$$

$$\Rightarrow \left(-\frac{1}{2\sqrt{2}}\right) \left(\frac{\beta}{\alpha}\right) = -1$$

$$\Rightarrow \beta = 2\sqrt{2}\alpha \quad \dots(1)$$

$$\therefore OD = \left|\frac{-4}{\sqrt{1+8}}\right| = \frac{4}{3} \Rightarrow AO = \frac{8}{3}$$

$$\text{So } AD = \frac{8}{3} + \frac{4}{3} = 4$$

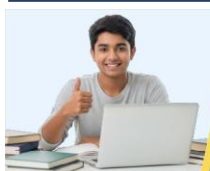
$$\Rightarrow \frac{|\alpha + 2\sqrt{2}\beta - 4|}{3} = 4 \Rightarrow \alpha = \frac{16}{9} \text{ or } -\frac{8}{9}$$

$$\left\{ \begin{array}{l} \because A(\alpha, \beta) \text{ \& } (0,0) \text{ lies on same} \\ \text{side of given line} \end{array} \right\}$$

$$\therefore (\alpha, \beta) = \left(\frac{16}{9}, \frac{32\sqrt{2}}{9}\right); \text{ (Rejected)}$$

$$\text{so } (\alpha, \beta) = \left(-\frac{8}{9}, -\frac{16\sqrt{2}}{9}\right)$$

$$= \left\lfloor \alpha + \sqrt{2}\beta \right\rfloor = \left\lfloor \frac{-8 - 32}{9} \right\rfloor = 4$$



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7. Let $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. Let x be the number of 9-digit numbers formed using the digits of the set S such that only one digit is repeated and it is repeated exactly twice. Let y be the number of 9-digit numbers formed using the digits of the set S such that only two digits are repeated and each of these is repeated exactly twice. Then,

- (1) $29x = 5y$ (2) $45x = 7y$
(3) $21x = 4y$ (4) $56x = 9y$

Ans. (3)

Sol. $S = \{1, 2, 3, \dots, 9\}$

$$x = {}^9C_1 \cdot {}^8C_7 \times \frac{9!}{2} = \frac{9 \times 8 \times 9!}{2}$$

$$y = {}^9C_2 \cdot {}^7C_5 \times \frac{9!}{2! \times 2!} = \frac{9 \times 8}{2} \times \frac{7 \times 6}{2} \times \frac{9!}{2! \times 2!}$$

$$\Rightarrow \frac{x}{y} = \frac{4}{21}$$

$$21x = 4y$$

8. Let $S = \{x^3 + ax^2 + bx + c : a, b, c \in \mathbb{N} \text{ and } a, b, c \leq 20\}$ be a set of polynomials. Then the number of polynomials in S , which are divisible by $x^2 + 2$, is

- (1) 20 (2) 6
(3) 120 (4) 10

Ans. (4)

Sol. $x^3 + ax^2 + bx + c = (x^2 + 2)\left(x + \frac{c}{2}\right)$

$$x^2 : a = \frac{c}{2}$$

$$x : b = 2$$

$$b = 2, a = \frac{c}{2} \quad c \in \{2, 4, \dots, 20\}$$

Number of polynomials in 'S' will be 10.

9. A bag contains 10 balls out of which k are red and $(10 - k)$ are black, where $0 \leq k \leq 10$. If three balls are drawn at random without replacement and all of them are found to be black, then the probability that the bag contains 1 red and 9 black balls is :

- (1) $\frac{7}{11}$ (2) $\frac{7}{55}$
(3) $\frac{7}{110}$ (4) $\frac{14}{55}$

Ans. (4)

$$\begin{aligned} \text{Sol. Probability} &= \frac{{}^1C_0 \cdot {}^9C_3}{\sum_{k=0}^{10} {}^kC_0 \cdot {}^{10-k}C_3} \\ &= \frac{{}^9C_3}{{}^{10}C_3 + {}^9C_3 + {}^8C_3 + \dots + {}^3C_3} \\ &= \frac{{}^9C_3}{{}^{11}C_4} = \frac{14}{55} \end{aligned}$$

10. Ans. ()

$$\text{Sol. } \frac{\beta - \alpha}{\alpha\beta} = \frac{1}{3}, \quad \alpha + \beta = \frac{\lambda + 3}{\lambda}, \quad \alpha\beta = \frac{3}{\lambda}$$

$$\beta - \alpha = \frac{\alpha\beta}{3} = \frac{1}{\lambda}$$

on squaring

$$\alpha^2 + \beta^2 - 2\alpha\beta = \frac{1}{\lambda^2} \quad \dots\dots\dots(1)$$

$$\alpha^2 + \beta^2 + 2\alpha\beta = \frac{(\lambda + 3)^2}{\lambda^2} \quad \dots\dots\dots(2)$$

$$(2) - (1) \quad 4\alpha\beta = \frac{(\lambda + 3)^2 - 1}{\lambda^2}$$

$$\frac{12}{\lambda} = \frac{\lambda^2 + 6\lambda + 8}{\lambda^2}$$

$$\Rightarrow \lambda^2 - 6\lambda^2 + 8\lambda = 0$$

$$\Rightarrow \lambda = 0, 2, 4$$

Sum of possible values of λ is = 6



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10. If α, β , where $\alpha < \beta$, are the roots of the equation

$$\lambda x^2 - (\lambda + 3)x + 3 = 0 \text{ such that } \frac{1}{\alpha} - \frac{1}{\beta} = \frac{1}{3}, \text{ then}$$

the sum of all possible values of λ is :

- (1) 6 (2) 2
(3) 4 (4) 8

Ans. (1)

Sol. $\frac{\beta - \alpha}{\alpha\beta} = \frac{1}{3}, \alpha + \beta = \frac{\lambda + 3}{\lambda}, \alpha\beta = \frac{3}{\lambda}$

$$\beta - \alpha = \frac{\alpha\beta}{3} = \frac{1}{\lambda}$$

on squaring

$$\alpha^2 + \beta^2 - 2\alpha\beta = \frac{1}{\lambda^2} \dots\dots\dots(1)$$

$$\alpha^2 + \beta^2 + 2\alpha\beta = \frac{(\lambda + 3)^2}{\lambda^2} \dots\dots\dots(2)$$

$$(2) - (1) \quad 4\alpha\beta = \frac{(\lambda + 3)^2 - 1}{\lambda^2}$$

$$\frac{12}{\lambda} = \frac{\lambda^2 + 6\lambda + 8}{\lambda^2}$$

$$\Rightarrow \lambda^2 - 6\lambda^2 + 8\lambda = 0$$

$$\Rightarrow \lambda = 0, 2, 4$$

Sum of possible values of λ is = 6

11. If $\int \left(\frac{1 - 5\cos^2 x}{\sin^5 x \cos^2 x} \right) dx = f(x) + C$ where C is the

constant of integration, then $f\left(\frac{\pi}{6}\right) - f\left(\frac{\pi}{4}\right)$ is equal to

- (1) $\frac{1}{\sqrt{3}}(26 + \sqrt{3})$ (2) $\frac{4}{\sqrt{3}}(8 - \sqrt{6})$
(3) $\frac{1}{\sqrt{3}}(26 - \sqrt{3})$ (4) $\frac{2}{\sqrt{3}}(4 + \sqrt{6})$

Ans. (2)

Sol. $\int \frac{dx}{\sin^5 x \cos^2 x} - 5 \int \frac{dx}{\sin^5 x}$

$$\int \frac{\sec^2 x dx}{\sin^5 x} - 5 \int \frac{dx}{\sin^5 x}$$

By IBP

$$= \frac{\tan x}{\sin^5 x} - \int -\frac{5}{\sin^6 x} \cdot \cos x \cdot \tan x dx - 5 \int \frac{dx}{\sin^5 x}$$

$$= \frac{\tan x}{\sin^5 x} + c$$

$$f(x) = \frac{\tan x}{\sin^5 x}$$

$$f\left(\frac{\pi}{6}\right) - f\left(\frac{\pi}{4}\right) = \frac{2^5}{\sqrt{3}} - (\sqrt{2})^5 = 4\sqrt{2} - \frac{32}{\sqrt{3}}$$

$$= \frac{32}{\sqrt{3}} - 4\sqrt{2}$$

$$= \frac{4}{\sqrt{3}}(8 - \sqrt{6})$$

12. Let f be a polynomial function such that $f(x^2 + 1) = x^4 + 5x^2 + 2$, for all $x \in \mathbb{R}$.

Then $\int_0^3 f(x) dx$ is equal to

- (1) $\frac{41}{3}$ (2) $\frac{33}{2}$
(3) $\frac{27}{2}$ (4) $\frac{5}{3}$

Ans. (2)

Sol. $\because f(x^2 + 1) = x^4 + 5x^2 + 2$
{put $x^2 + 1 = t$ }

$$\Rightarrow f(t) = (t - 1)^2 + 5(t - 1) + 2$$

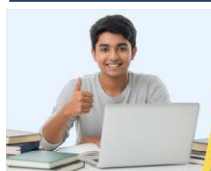
$$\Rightarrow f(t) = t^2 + 3t - 2$$

$$\text{Now, } \int_0^3 f(t) dt = \int_0^3 (t^2 + 3t - 2) dt$$

$$\left[\frac{t^3}{3} + \frac{3t^2}{2} - 2t \right]_0^3$$

$$\left[\frac{27}{3} + \frac{27}{2} - 6 \right]$$

$$= \frac{33}{2}$$



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13. The area of the region

$$R = \{(x, y) : xy \leq 8, 1 \leq y \leq x^2, x \geq 0\} \text{ is}$$

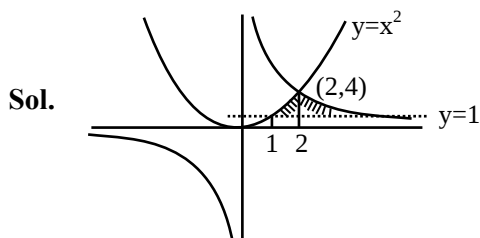
(1) $\frac{1}{3}(49 \log_e(2) - 15)$

(2) $\frac{2}{3}(20 \log_e(2) + 9)$

(3) $\frac{2}{3}(24 \log_e(2) - 7)$

(4) $\frac{1}{3}(40 \log_e(2) + 27)$

Ans. (3)



$$A = \int_1^2 (x^2 - 1) dx + \int_2^8 \left(\frac{8}{x} - 1\right) dx$$

$$A = 8 \log_e 4 - \frac{14}{3} = 16 \log_e 2 - \frac{14}{3}$$

$$= \frac{2}{3}(24 \log_e 2 - 7)$$

14. The value of

$$\lim_{x \rightarrow 0} \frac{\log_e(\sec(ex) \cdot \sec(e^2x) \cdot \dots \cdot \sec(e^{10}x))}{e^2 - e^{2 \cos x}}$$

is equal to

(1) $\frac{(e^{10} - 1)}{2e^2(e^2 - 1)}$

(2) $\frac{(e^{20} - 1)}{2e^2(e^2 - 1)}$

(3) $\frac{(e^{20} - 1)}{2(e^2 - 1)}$

(4) $\frac{(e^{10} - 1)}{2(e^2 - 1)}$

Ans. (3)

Sol. $\Rightarrow \lim_{x \rightarrow 0} \frac{\ln(\sec(ex)) + \ln(\sec(e^2x)) + \dots + \ln(\sec(e^{10}x))}{e^{2 \cos x} \left(\frac{e^{2-2 \cos x} - 1}{2 - 2 \cos x} \right) \times \frac{2 - 2 \cos x}{x^2} \times x^2}$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{\ell n(\sec(ex)) + \ell n(\sec(e^2x)) + \dots + \ell n(\sec(e^{10}x))}{e^2 x^2}$$

Using L'H rule

$$\Rightarrow \lim_{x \rightarrow 0} \frac{e \tan ex + e^2 \tan e^2x + \dots + e^{10} \tan e^{10}x}{2e^2x}$$

$$\Rightarrow \frac{1}{2e^2} [e^2 + e^4 + e^6 + \dots + e^{20}]$$

$$\Rightarrow \frac{1}{2} \frac{e^2((e^2)^{10} - 1)}{e^2(e^2 - 1)}$$

$$\Rightarrow \frac{1}{2} \frac{(e^{20} - 1)}{(e^2 - 1)}$$

15. The mean and variance of 10 observations are 9 and 34.2, respectively. If 8 of these observations are 2, 3, 5, 10, 11, 13, 15, 21, then the mean deviation about the median of all the 10 observations is

(1) 5 (2) 4

(3) 6 (4) 7

Ans. (1)

Sol. $\frac{2+3+5+10+11+13+15+21+a+b}{10} = 9$

$$\frac{80+a+b}{10} = 9 \Rightarrow a+b = 10 \dots (1)$$

$$\frac{\sum x_i^2}{10} - \left(\frac{\sum x_i}{10} \right)^2 = 34.2$$

$$\frac{2^2+3^2+5^2+10^2+11^2+13^2+15^2+21^2+a^2+b^2}{10} - (9)^2 = 34.2$$

$$1094 + a^2 + b^2 - 810 = 342$$

$$a^2 + b^2 = 58 \dots (2)$$

$$a = 7, b = 3$$

$$\text{or } a = 3, b = 7$$

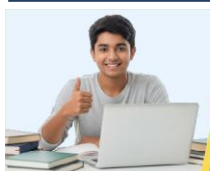
$$\text{Number} \rightarrow 2, 3, 5, 7, 10, 11, 13, 15, 21$$

$$\text{Mean} = \frac{7+10}{2} = 8.5$$

$$\text{M.D} = \frac{6.5+5.5+5.5+3.5+1.5+1.5+2.5+4.5+6.5+12.5}{10}$$

$$= \frac{50}{5}$$

$$= 10$$



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16. Let A, B and C be three 2×2 matrices with real entries such that $B = (I + A)^{-1}$ and $A + C = I$. If

$$BC = \begin{bmatrix} 1 & -5 \\ -1 & 2 \end{bmatrix} \text{ and } CB \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 12 \\ -6 \end{bmatrix}, \text{ then}$$

$x_1 + x_2$ is

- (1) 2 (2) 0
(3) -2 (4) 4

Ans. (2)

Sol. $B = (I + A)^{-1}$, $A + C = I$

$$\Rightarrow B(I + A) = (I + A)B = I$$

$$\Rightarrow B + BA = B + AB$$

$$\Rightarrow B + B(I - C) = B + (I - C)B$$

$$\Rightarrow 2B - BC = 2B - CB$$

$$\Rightarrow BC = CB$$

$$\therefore CB \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & -5 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 12 \\ -6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & -5 \\ -1 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 12 \\ -6 \end{bmatrix} = -\frac{1}{3} \begin{bmatrix} 2 & 5 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 32 \\ -6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \end{bmatrix} \therefore x_1 + x_2 = 0$$

17. The common difference of the A.P.: $a_1, a_2, \dots, a_n$ is 13 more than the common difference of the A.P.: $b_1, b_2, \dots, b_n$. If $b_{31} = -277$, $b_{43} = -385$ and $a_{78} = 327$, then a_1 is equal to

- (1) 21 (2) 24
(3) 19 (4) 16

Ans. (3)

Sol. Let common difference of A.P.'s are d_1 & d_2

$$\therefore d_1 = 13 + d_2$$

$$b_1 + 30d_2 = -277 \quad \dots(1)$$

$$b_1 + 42d_2 = -385 \quad \dots(2)$$

$$\text{By (2) - (1)}$$

$$12d_2 = -108$$

$$d_2 = -9$$

$$\therefore d_1 = 4$$

$$\text{Now } a_{78} = 327$$

$$\Rightarrow a_1 + 77d_1 = 327$$

$$\Rightarrow a_1 + 308 = 327$$

$$a_1 = 19$$

18. If the distances of the point $(1, 2, a)$ from the line

$$\frac{x-1}{1} = \frac{y}{2} = \frac{z-1}{1} \text{ along the lines}$$

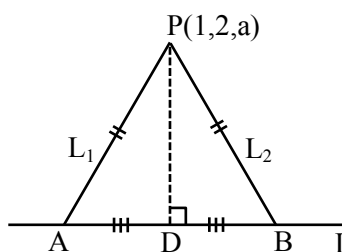
$$L_1: \frac{x-1}{3} = \frac{y-2}{4} = \frac{z-a}{b} \text{ and}$$

$$L_2: \frac{x-1}{1} = \frac{y-2}{4} = \frac{z-a}{c} \text{ are equal,}$$

then $a + b + c$ is equal to

- (1) 7 (2) 5
(3) 6 (4) 4

Ans. (1)



Sol.

$$L: \frac{x-1}{1} = \frac{y}{2} = \frac{z-1}{1}$$

$$L_1: \frac{x-1}{3} = \frac{y-2}{4} = \frac{z-a}{b} = \lambda$$

$$L_2: \frac{x-1}{1} = \frac{y-2}{4} = \frac{z-a}{c} = \mu$$

$$\text{Let } A(3\lambda + 1, 4\lambda + 2, b\lambda + a)$$

It lies on L

$$\therefore \frac{3\lambda}{1} = \frac{4\lambda + 2}{2} = \frac{b\lambda + a - 1}{1}$$

$$\Rightarrow \boxed{\lambda = 1} \text{ and } a + b - 1 = 3$$

$$\Rightarrow A(4, 6, 4), a + b = 4 \quad \dots(1)$$

$$\text{Let } B(\mu + 1, 4\mu + 2, c\mu + a)$$

It also lies on L

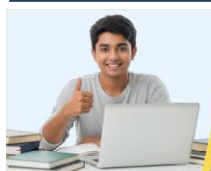
$$\frac{\mu}{1} = \frac{4\mu + 2}{2} = \frac{c\mu + a - 1}{1}$$

$$\Rightarrow 2\mu = 4\mu + 2$$

$$\Rightarrow \boxed{\mu = -1}$$

$$a - c - 1 = -1$$

$$\Rightarrow \boxed{a = c} \quad \dots(2) \text{ \& } B(0, -2, 0)$$



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also $PA = PB$, $P(1, 2, a)$, $A(4, 6, 4)$

$$\Rightarrow 9 + 16 + (a - 4)^2 = 1 + 16 + a^2$$

$$\Rightarrow 16 + 8 = 8a$$

$$\boxed{a=3} \quad \therefore c = 3, b = 1$$

$$\therefore \boxed{a + b + c = 7}$$

19. For three unit vectors $\vec{a}, \vec{b}, \vec{c}$ satisfying

$$|\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2 = 9 \text{ and}$$

$$|2\vec{a} + k\vec{b} + k\vec{c}| = 3,$$

the positive value of k is :

$$(1) 3 \quad (2) 6$$

$$(3) 4 \quad (4) 5$$

Ans. (4)

Sol. $|\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2 = 9$

$$\Rightarrow \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = -\frac{3}{2}$$

$$\Rightarrow \vec{a} + \vec{b} + \vec{c} = 0 \Rightarrow \vec{b} + \vec{c} = -\vec{a}$$

$$|2\vec{a} + k(\vec{b} + \vec{c})| = 3$$

$$|\vec{a}(2 - k)| = 3$$

$$K = 5 \text{ or } -1$$

Positive value of k is 5

20. Let $y = y(x)$ be the solution of the differential equation

$$x \frac{dy}{dx} - \sin 2y = x^3(2 - x^3) \cos^2 y, \quad x \neq 0.$$

If $y(2) = x$, then $\tan(y(1))$ is equal to

$$(1) \frac{3}{4} \quad (2) \frac{7}{4}$$

$$(3) -\frac{7}{4} \quad (4) -\frac{3}{4}$$

Ans. (2)

Sol. $x \frac{dy}{dx} - \sin 2y = x^3(2 - x^3) \cos^2 y$

$$\sec^2 y \frac{dy}{dx} - 2 \tan y \cdot \frac{1}{x} = x^2(2 - x^3)$$

$$\tan y = t \Rightarrow \sec^2 y \frac{dy}{dx} = \frac{dt}{dx}$$

$$\frac{dt}{dx} - \frac{2t}{x} = x^2(2 - x^3) \quad (\text{LDE})$$

$$\text{I.F.} = e^{\int -\frac{2}{x} dx} = e^{-2 \ln x} = \frac{1}{x^2}$$

$$\therefore \frac{t}{x^2} = \int \frac{1}{x^2} x^2(2 - x^3) dx + C$$

$$\frac{\tan y}{x^2} = 2x - \frac{x^4}{4} + C$$

$$y(2) = 0 \Rightarrow 0 = 4 - 4 + C \Rightarrow C = 0$$

$$\tan y = 2x^3 - \frac{1}{4}x^6$$

$$x = 1 \Rightarrow \tan y = 2 - \frac{1}{4} = \frac{7}{4} \Rightarrow (2)$$

SECTION-B

21. In a G.P., if the product of the first three terms is 27 and the set of all possible values for the sum of its first three terms is $\mathbb{R} - (a, b)$, then $a^2 + b^2$ is equal to ____.

Ans. (90)

Sol. Let first three terms of G.P. are $\frac{A}{r}, A, Ar$

$$\frac{A}{r} \cdot A \cdot Ar = 27$$

$$A = 3$$

$$3\left(\frac{1}{r} + 1 + r\right) = 3 + 3\left(r + \frac{1}{r}\right)$$

$$\text{We know, } r + \frac{1}{r} \geq 2 \text{ or } r + \frac{1}{r} \leq -2$$

$$S \in \mathbb{R} - (-3, 9)$$

$$a^2 + b^2 = 9 + 81 = 90$$



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22. For some $\theta \in \left(0, \frac{\pi}{2}\right)$, let the eccentricity and the length of the latus rectum of the hyperbola $x^2 - y^2 \sec^2 \theta = 8$ be e_1 and ℓ_1 , respectively, and let the eccentricity and the length of the latus rectum of the ellipse $x^2 \sec^2 \theta + y^2 = 6$ be e_2 and ℓ_2 , respectively. If $e_1^2 = e_2^2 (\sec^2 \theta + 1)$, then $\left(\frac{\ell_1 \ell_2}{e_1 e_2}\right) \tan^2 \theta$ is equal to _____.

Ans. (8)

Sol. $\frac{x^2}{8} - \frac{y^2}{8 \cos^2 \theta} = 1$, $e_1 = \sqrt{1 + \frac{8 \cos^2 \theta}{8}}$

$$\ell_1 = \frac{2b^2}{a} = \frac{2 \cdot (8 \cos^2 \theta)}{2\sqrt{2}}$$

$$\frac{x^2}{6} + \frac{y^2}{6 \cos^2 \theta} = 1$$
; $e_2 = \sqrt{1 - \frac{6 \cos^2 \theta}{6}} = \sin \theta$

$$\ell_2 = \frac{2b^2}{a} = \frac{2 \cdot 6 \cos^2 \theta}{\sqrt{6}}$$

$$e_1^2 = e_2^2 (1 + \sec^2 \theta)$$

$$1 + \cos^2 \theta = \sin^2 \theta \left(1 + \frac{1}{\cos^2 \theta}\right)$$

$$1 + \cos^2 \theta = \sin^2 \theta + \tan^2 \theta$$

Solving we get $\theta = \frac{\pi}{4}$

$$\ell_1 = 2\sqrt{2}$$

$$e_1 = \sqrt{\frac{3}{2}}$$

$$\ell_2 = \sqrt{6}$$

$$e_2 = \frac{1}{\sqrt{2}}$$

$$\left(\frac{\ell_1 \ell_2}{e_1 e_2}\right) \tan^2 \theta = 8 \quad (\text{By putting values})$$

23. If $k = \tan\left(\frac{\pi}{4} + \frac{1}{2} \cos^{-1}\left(\frac{2}{3}\right)\right) + \tan\left(\frac{1}{2} \sin^{-1}\left(\frac{2}{3}\right)\right)$ then the number of solutions of the equation $\sin^{-1}(kx - 1) = \sin^{-1} x - \cos^{-1} x$ is _____.

Ans. (1)

Sol. Let $\theta = \frac{1}{2} \sin^{-1} \frac{2}{3}$, then $\frac{1}{2} \cos^{-1} \frac{1}{3} = \left(\frac{\pi}{4} - \theta\right)$

$$k = \tan \theta + \cot \theta = \frac{1}{\sin \theta \cos \theta} = \frac{2}{\sin 2\theta}$$

$$k = \frac{2}{\frac{2}{3}} = 3$$

$$\sin^{-1}(3x - 1) = \sin^{-1} x - \cos^{-1} x$$

$$\sin^{-1}(3x - 1) = \frac{\pi}{2} - 2\cos^{-1} x$$

$$3x - 1 = \sin\left(\frac{\pi}{2} - 2\cos^{-1} x\right)$$

$$3x - 1 = 2x^2 - 1 \Rightarrow x = 0, \frac{3}{2} \text{ (rejected)}$$

No. of solution = 1

24. The value of $\sum_{r=1}^{20} \left(\left| \sqrt{\pi \left(\int_0^r x |\sin \pi x| dx \right)} \right| \right)$ is _____.

Ans. (210)

Sol. Let $I_r = \int_0^r x |\sin \pi x| dx \dots (1)$

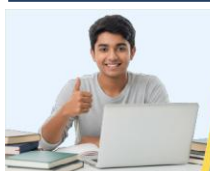
Apply King Property

$$= \int_0^r (r - x) |\sin \pi x| dx \dots (2)$$

By (1) + (2)

$$2I_r = \int_0^r r |\sin \pi x| dx \Rightarrow I_r = \frac{r}{2} \int_0^r |\sin \pi x| dx$$

$$I_1 = \frac{1}{2} \int_0^1 |\sin \pi x| dx = \frac{1}{2\pi} \int_0^\pi |\sin t| dt = \frac{1}{2\pi} (2)$$



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$$I_2 = \frac{2}{2} \int_0^2 |\sin \pi x| dx = \frac{2}{2\pi} \int_0^{2\pi} |\sin t| dt = \frac{2}{2\pi} (4)$$

$$S = \sqrt{\pi \cdot \frac{1}{2\pi} \cdot 2} + \sqrt{\pi \cdot \frac{2}{2\pi} \cdot 4} + \sqrt{\pi \cdot \frac{3}{2\pi} \cdot 6} + \dots + \sqrt{\pi \cdot \frac{20}{2\pi} \cdot (2 \cdot 20)}$$

$$= 1 + 2 + 3 + \dots + 20$$

$$= \frac{20 \times 21}{2} = 210$$

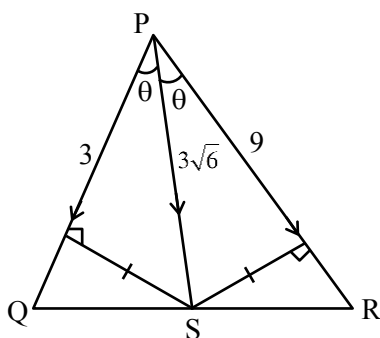
25. Let PQR be a triangle such that $\overrightarrow{PQ} = -2\hat{i} - \hat{j} + 2\hat{k}$ and $\overrightarrow{PR} = a\hat{i} + b\hat{j} - 4\hat{k}$, $a, b \in \mathbb{Z}$. Let S be the point on QR, which is equidistant from the lines PQ and PR. If $|\overrightarrow{PR}| = 9$ and $\overrightarrow{PS} = \hat{i} - 7\hat{j} + 2\hat{k}$, then the value of $3a - 4b$ is ____.

Ans. (37)

Sol. $\overrightarrow{PQ} = -2\hat{i} - \hat{j} + 2\hat{k}$

$$\overrightarrow{PR} = a\hat{i} + b\hat{j} - 4\hat{k} \quad (a, b \in \mathbb{Z})$$

$$\overrightarrow{PS} = \hat{i} - 7\hat{j} + 2\hat{k}$$



$$|\overrightarrow{PR}| = 9$$

$$a^2 + b^2 + 16 = 81$$

$$a^2 + b^2 = 65 \quad \dots(1)$$

$$\cos \theta = \frac{\overrightarrow{PQ} \cdot \overrightarrow{PS}}{|\overrightarrow{PQ}| |\overrightarrow{PS}|}$$

$$= \frac{-2 + 7 + 4}{3 \cdot 3\sqrt{6}} = \frac{9}{3 \cdot 3\sqrt{6}}$$

$$= \frac{1}{\sqrt{6}}$$

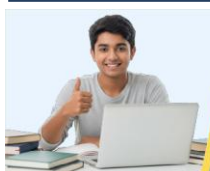
$$\frac{1}{\sqrt{6}} = \frac{\overrightarrow{PS} \cdot \overrightarrow{PR}}{|\overrightarrow{PS}| |\overrightarrow{PR}|} = \frac{a - 7b - 8}{3\sqrt{6} \cdot 9}$$

$$a - 7b = 35 \quad \dots(2)$$

From (1) & (2)

$$\Rightarrow a = 7, b = -4$$

$$\therefore 3a - 4b = 21 + 16 = 37$$



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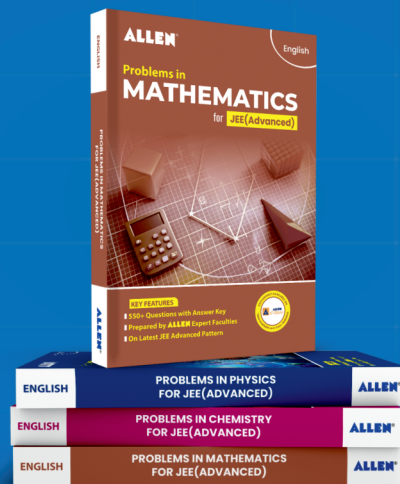
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